令和8年度入学
筑波大学大学院入学試験
8月実施
理工情報生命学術院　数理物質科学研究群
応用理工学学位プログラム
電子・物理工学サブプログラム　試験問題

専門科目

注意事項（選択、解答についての必要な指示）

1. 問題は 6 題 (Problems 1–6)あり、このうち Problem 1 は必ず解答しなさい。
   また Problem 2 から Problem 6 までの中から 3 題を選び解答しなさい。
   ただし、Problem 2 または Problem 3 のどちらかは必ず解答すること。
2. 日本語または英語で解答すること。
3. Problem 1 は Part(1)と Part(2)から成り、それぞれ別の問題用紙に記載されています。
4. 4 題を超えて解答したときは、すべての答案を無効とします。
5. 答案用紙全てに受験番号を記入すること。
6. 答案用紙は全部で 5 枚あります。
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   Problem 2 から Problem 6 の選択問題は、残り 3 枚の答案用紙を使って 1 題を 1 枚の答案用紙に解答しなさい。この場合、各答案用紙の左上に必ず問題番号を記入しなさい。紙面が足りないときは裏面を使うこと。
7. 答案用紙のホチキス針は取り外さないこと。

Entrance Examination for 2026 Enrollment
August Selection Process

Master' s Program in Engineering Sciences
Subprogram in Applied Physics

Degree Program in Pure and Applied Sciences
Graduate School of Science and Technology
University of Tsukuba

Specialized Subject

Notes:

1. There are six problems given. Problem 1 is COMPULSORY. From Problems 2 to 6, select THREE problems and solve them. Either or both Problems 2 and 3 have to be selected.
2. Answers must be in Japanese or English.
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Applied Physics

Problem 1 (140 points) Mathematics [数学] Part (1)

Note: Problem 1 has two parts, (1) and (2). Answer all of them.

(1) Answer the following questions (a), (b), (c), and (d). Let x, y, r, θ, and λ be real numbers [実数], and z be a complex number [複素数].

(a) Evaluate the following limit.

$$\lim_{x \to 0} \frac{e^{-5x} + 5x - 1}{\sin^2 x}.$$

(b) Evaluate the following double integral [2 重積分] over the given domain D by using a change of variables [変数変換], $x = r\cos\theta$, $y = 2r\sin\theta$.

$$\iint_D y\,dxdy, \quad D = \left\{(x,y) \middle| x^2 + \left(\frac{y}{2}\right)^2 \le 1,\ x \ge 0,\ y \le 0\right\}.$$

(c) Answer the following questions concerning two bivariate functions [2 変数関数].

$$F(x,y) = xy, \quad G(x,y) = x^2 + y^2 - 4\,.$$

(i) Find the first-order partial derivatives [第 1 次偏導関数] of $F(x,y)$ and $G(x,y)$, i.e., $F_x(x,y)$, $F_y(x,y)$, $G_x(x,y)$, and $G_y(x,y)$.

(ii) Find all the solutions (x,y) to the following simultaneous equations [連立方程式], where $G_x(x,y)^2 + G_y(x,y)^2 \neq 0$, and λ is the undetermined multiplier [未定乗数].

$$\begin{cases} F_x(x,y) - \lambda G_x(x,y) = 0 \\ F_y(x,y) - \lambda G_y(x,y) = 0 \\ G(x,y) = 0 \end{cases}$$

(d) Answer the following questions concerning the complex function [複素関数] $f(z)$.

$$f(z) = -\frac{1}{z^2 + z}.$$

(i) Find the Laurent series expansion [ローラン級数展開] of $f(z)$ in $0 < |z+1| < 1$.

(ii) Evaluate the following complex integral along a circular path [円経路] C oriented counterclockwise [反時計回り].

$$\int_C f(z)\,dz, \qquad C: |z+1| = \frac{1}{2}.$$

(continued to part (2) on the next page)

Applied Physics

Problem 1 (140 points) Mathematics [数学] Part (2)

Note: Problem 1 has two parts, (1) and (2). Answer all of them.

(2) Consider the 2×2 matrix $A = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}$ and the 4×4 matrix $B = \begin{pmatrix} O & A \\ A & O \end{pmatrix}$.

Here, a is a positive number and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Answer the following questions.

(a) Find the eigenvalues λ_1 and λ_2 ($\lambda_1 \geq \lambda_2$) of the matrix A.

(b) Find the normalized eigenvectors (正規化された固有ベクトル) $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ corresponding to the eigenvalues λ_1 and λ_2, respectively such that the first component of each of these vectors is nonnegative.

(c) Find the orthogonal matrix (直交行列) P and P^{-1} that diagonalize A (Aを対角化する) in such a way that $P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$.

(d) Find the product of 4×4 matrices QBQ where $Q = \frac{1}{\sqrt{2}}\begin{pmatrix} E & E \\ E & -E \end{pmatrix}$ and $E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Applied Physics

Problem 2 (120 points) Mechanics

Consider the motion of a pendulum [振り子] consisting of a wire of negligible mass fixed at one end with a uniform sphere of mass M and radius a, as shown in Fig. 2-1. The other end of the wire is fixed at the origin O. Let ℓ be the distance between the origin O and the center of gravity [重心] of the sphere, I be the moment of inertia [慣性モーメント] about the axis of rotation of the pendulum, θ be the angle between the wire and the vertical direction, and g be the magnitude of the gravitational acceleration [重力加速度].

First, ignore the air resistance [空気抵抗] and answer the following questions.

(1) Express I by $M, \ell,$ and a. Note that the moment of inertia of a uniform sphere (mass M and radius a) about any axis through its center is $\frac{2}{5}Ma^2$.

(2) Find the equation of motion [運動方程式] for the pendulum.

(3) Find the period [周期] for small oscillations ($|\theta| \ll 1$).

(4) Given that θ_m ($< \pi/2$) is the maximum swing angle of the pendulum, is the period of the pendulum constant regardless of θ_m, or is it an increasing or decreasing function of θ_m ? Answer with the physical basis [物理的根拠].

Next, consider the force of the air resistance acting on the sphere. Let the magnitude of the force of the air resistance be $2M\gamma\ell\dot{\theta}$ ($\gamma > 0$). Assuming that $\ell \gg a$ and the pendulum experiences small oscillations ($|\theta| \ll 1$), answer the following questions.

(5) Find the equation of motion for the pendulum.

(6) When $\gamma < \sqrt{g/\ell}$, the pendulum undergoes damped oscillation [減衰振動]. Find the period of the oscillation.

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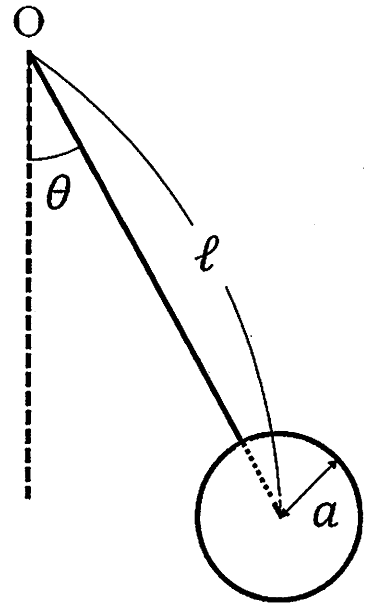

Figure 2-1

Applied Physics

Problem 3 (120 points) Electromagnetism [電磁気学]

Suppose that a point charge $q(>0)$ is placed at the origin O in three-dimensional space, and the point charge is enclosed by a conducting spherical shell centered at O with inner radius a and outer radius $b\ (>a)$, as shown in Fig. 3-1. The thickness of the conducting spherical shell is $b-a$ and it is assumed that the conducting shell has no net charge. Let ε_0 denote the vacuum permittivity, and regions $0<r<a$ and $r>b$ are assumed to be vacuum, where r is the distance from O. The electrostatic potential is set to zero at infinity. Answer the following questions concerning the electric field and electrostatic potential of a point at the distance r from O.

(1) Express the strength of the electric field in the inner region of the conducting spherical shell $(0<r<a)$ in terms of $q,\ r$, and ε_0.

(2) Find the strength of the electric field in the conducting spherical shell's interior $(a<r<b)$ excluding the surface.

(3) Express the strength of the electric field in the outer region of the conducting spherical shell $(r>b)$ in terms of $q,\ r$, and ε_0.

(4) (a) Express the electrostatic potential in the outer region of the conducting spherical shell $(r>b)$ in terms of $q,\ r$, and ε_0.

(b) Express the electrostatic potential in the conducting spherical shell's interior $(a\le r\le b)$ including the surface in terms of $q,\ \varepsilon_0$, and b.

(c) Express the electrostatic potential in the inner region of the conducting spherical shell $(0<r<a)$ in terms of $q,\ r,\ \varepsilon_0, a$, and b.

(d) Draw the graph of the electrostatic potential in the whole region $(0<r<\infty)$.

Next, suppose that a point charge $q(>0)$ is placed at the origin O in three-dimensional space. The charge is enclosed by a dielectric spherical shell centered at O with inner radius a and outer radius $b\ (>a)$, as shown in Fig. 3-1. The thickness of the dielectric spherical shell is $b-a$, the permittivity of the dielectric shell is $\varepsilon(>\varepsilon_0)$, and it is assumed that the dielectric shell has no net charge. Answer the following questions concerning the electric flux density, the electric field, and electric charge density of a point at a distance r from O. The regions $r<a$ and $r>b$ are assumed to be vacuum.

(*continued to the next page*)

(5) Express the strength of the electric flux density and that of the electric field in the dielectric spherical shell's interior $(a < r < b)$ excluding the surface in terms of necessary ones among q, r, ε_0, and ε.

(6) Express the surface charge density induced on the inner surface $(r = a)$ of the dielectric spherical shell in terms of q, a, ε_0, and ε. It is assumed that the induced surface charge density is uniform.

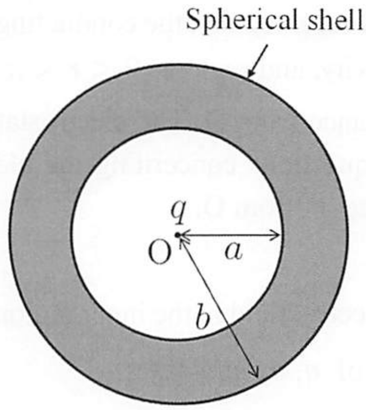

Figure 3-1

Applied Physics

Problem 4 (120 points) Quantum Mechanics

Answer the following questions regarding the electronic states [電子状態] in a hydrogen atom. We represent the distance from the nucleus [原子核] by r, the reduced mass [換算質量] of the electron by m, Bohr radius by a_0 and the reduced Planck constant (Dirac constant) by $\hbar$. Then, the potential of the system is represented as:

$$V(r) = -\frac{\hbar^2}{m}\frac{1}{a_0 r}.$$

Use the following relationships between the x, y, z Cartesian coordinate system and the r, θ, ϕ spherical coordinate system.

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\hat{\Lambda},$$

$$\hat{\Lambda} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.$$

(1) Write down the time-independent Schrödinger equations for the wave functions of the electron, both in the x, y, z Cartesian coordinate system $\varphi(x, y, z)$ and in the r, θ, ϕ spherical coordinate system $\varphi(r, \theta, \phi)$, with the eigen energy [固有エネルギー] ε. Do not use Δ, $\hat{\Lambda}$, or $V(r)$ in the answer. In addition to the coordinates either x, y, z, or r, θ, ϕ, only the constants $\hbar, m$ and a_0 are allowed to be included.

(2) The wave function for the ground state [基底状態] of this system φ_1 can be represented as $\varphi_1(r, \theta, \phi) = Ae^{-r/a_0}$ with a normalization constant [規格化定数] A. Show that this function satisfies the Schrödinger equation in the spherical coordinate system and find its eigen energy ε_1.

(3) Find the probability [確率] to observe the electron in the ground state in the range of $r \leq a_0$.

Hint: $\int_0^a r^2 e^{-r} dr = 1 - (1 + a)e^{-a}$.

(continued to the next page)

(4) The first excited state [第一励起状態] of this system is quadruply degenerated [四重に縮退]. Among them, the wave functions for the three p orbitals [p 軌道] can be represented as:

$$\varphi_2^{-1} = Bre^{-r/2a_0}e^{-i\phi}\sin\theta\,,$$
$$\varphi_2^{0} = Bre^{-r/2a_0}\cos\theta\,,$$
$$\varphi_2^{+1} = Bre^{-r/2a_0}e^{+i\phi}\sin\theta\,,$$

where B is a normalization constant. Show that the eigenvalue [固有値] of the angular momentum squared operator $\hat{l^2}$ is $2\hbar^2$ for all of the three, and the eigenvalues of the z-component of the angular momentum $\hat{l}_z$ are $-\hbar, 0$, and $+\hbar$. Note that the corresponding operators are $\hat{l^2} = -\hbar^2\hat{\Lambda}$ and $\hat{l}_z = -i\hbar\partial/\partial\phi$. Here i is an imaginary unit.

(5) We define the following three wave functions by the functions given in (4):

$$\varphi_2^{x} = (\varphi_2^{+1} + \varphi_2^{-1})/\sqrt{2}\,,$$
$$\varphi_2^{y} = (\varphi_2^{+1} - \varphi_2^{-1})/\sqrt{2}i\,,$$
$$\varphi_2^{z} = \varphi_2^{0}\,.$$

Represent these functions in x, y, z coordinate system, without using r, θ, ϕ. Note that, when B is a real number [実数], these functions become real functions [実関数].

(6) Show that φ_2^{x} is not an eigenfunction [固有関数] of $\hat{l}_z$.

Applied Physics

Problem 5 (120 points) Optics [光学]

(1) There are a concave lens [凹レンズ] and a convex lens [凸レンズ] with focal lengths [焦点距離] of $-2f$ and f, respectively ($f > 0$) as shown in Figs. 5-1 and 5-2. They are assumed to be thin lens and satisfy the paraxial approximation [近軸近似]. Suppose that there is an object located at a distance a_1 (> 0) to the left of the concave lens. Answer the following questions.

First, as for Fig. 5-1, an image is formed at a distance b_1 from the concave lens.

(a) Express b_1 in terms of a_1 and f. Note that b_1 is positive when it is on the right side of the concave lens.

(b) Explain whether the image is real or virtual.

Next, as shown in Fig. 5-2, the convex lens is placed at a distance L (> 0) from the concave lens. An image is formed at a distance b_2 from the convex lens.

(c) Express b_2 in terms of a_1, f, and L. Note that b_2 is positive when it is on the right side of the convex lens.

(d) Find the optical magnification [光学倍率] of the lens pair when $L = f$.

(e) Find the combined focal length [合成焦点距離] of the lens pair when $L = 0$.

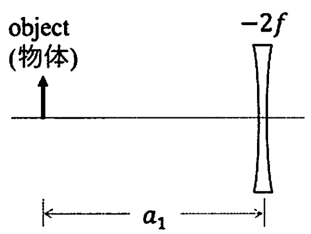

Figure 5-1

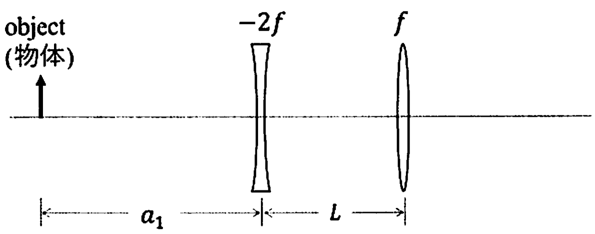

Figure 5-2

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(2) As shown in Fig. 5-3, monochromatic light [単色光] from a light source passes through a slit and diffracts. The light further diffracts through two slits S_1 and S_2, which are separated by a distance of $2a$. The light shows an interference fringe [干渉縞] on a screen placed at a distance L. The electric fields at position x on the screen from the slits S_1 and S_2 are expressed as:

$$E_1 = E_0 \exp(ikr_1 - i\omega t + i\phi_1).$$
$$E_2 = E_0 \exp(ikr_2 - i\omega t + i\phi_2).$$

Here, E_0 is the amplitude, k is the wave number, ω is the angular frequency of light, and ϕ_1 and ϕ_2 are the initial phases [初期位相]. r_1 and r_2 are the distances from S_1 and S_2 to x, respectively. Assuming that L is sufficiently large so that the following approximations are available.

$$r_1 = \sqrt{L^2 + (a-x)^2} \sim L + \frac{a^2 - 2ax}{2L}, \quad r_2 = \sqrt{L^2 + (a+x)^2} \sim L + \frac{a^2 + 2\,ax}{2L}.$$

The intensity $I(x)$ of the interference fringe at position x is given by

$$I(x) = |E_1 + E_2|^2.$$

Answer the following questions.

(a) Express $r_2 - r_1$ by using $a, x,$ and L.

(b) When $\phi_1 = \phi_2 = 0$, express $I(x)$ and the period of the interference fringe using E_0, k, a, L and x.

(c) When $\phi_2 = \phi_1 + \delta$, determine how much the antinode position [腹の位置] of the interference fringe shifts compared to the one in the case of $\phi_1 = \phi_2 = 0$.

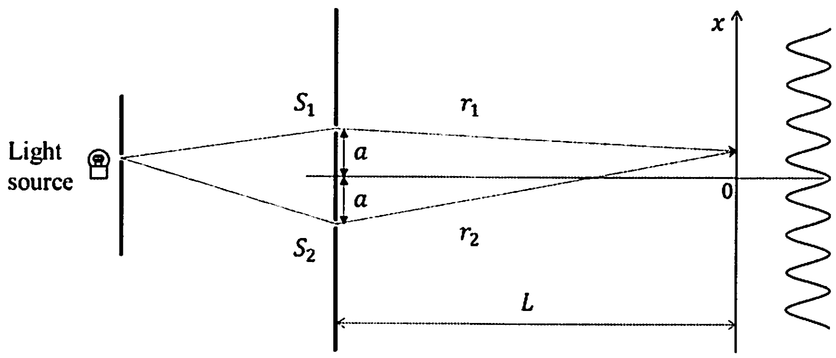

Figure 5-3

Applied Physics

Problem 6 (120 points) Semiconductor Physics

Consider an ideal metal/oxide/p-type semiconductor (MOS) structure, as shown in Fig. 6-1, in its steady state at room temperature with no fixed charge in the oxide and at the interfaces. Acceptor impurities (concentration: N_A) are uniformly distributed and are fully ionized. Let the elementary charge, the permittivity (誘電率) of the semiconductor, the permittivity of the oxide, the conduction band minimum, the valence band maximum, the Fermi level, the intrinsic Fermi level, and the thickness of the oxide be q, ε_S, ε_{OX}, E_C, E_V, E_F, E_i, T_{OX}, respectively. The work functions (仕事関数) of the metal and the semiconductor are equal, and the voltage V_G is applied to the metal with respect to the semiconductor. V_{OX} and ϕ_S are the voltage drop across the oxide and the surface potential (表面電位) of the semiconductor, respectively. Assuming that the complete depletion approximation (完全空乏近似) is applicable, answer the following questions.

(1) Draw the band diagram of the MOS structure when $V_G = 0$. Indicate E_C, E_V, E_F, and E_i in the diagram.

(2) Draw the band diagrams of the MOS structure in accumulation (蓄積) regime, weak inversion (弱反転) regime, and strong inversion (強反転) regime. Indicate E_C, E_V, E_F, E_i, and $q\phi_B$ in the diagrams, where $q\phi_B = E_i - E_F$. Also, show the relative position of E_i and E_F at the oxide/p-type semiconductor interface.

(3) What is the type of charges generated in the semiconductor in the three regimes mentioned in (2) ? The term "type of charges" refers to "hole", "electron", or "acceptor ion". Also, answer whether these charges are positive or negative.

(4) Let the depletion layer (空乏層) width be W_p, and solve the Poisson's equation (ポアソン方程式) and express ϕ_S with q, N_A, W_p, and ε_S for the weak inversion regime. Note that the electric field is zero at the depletion layer edge.

(5) Express the space charge (空間電荷) areal density Q_B in the semiconductor in the weak inversion regime with q, N_A, and W_p.

(6) Based on the relation that $V_G = \phi_S + V_{OX}$, and on the results in (4) and (5), express V_G for the weak inversion regime with q, N_A, ε_{OX}, W_p, and T_{OX}.

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(7) In the strong inversion regime, an increase in V_G results in a nearly equal increase in V_{OX}. Explain the reason.

(8) Explain the process of minority carrier generation, and describe how the minority carriers are collected in the inversion layer.

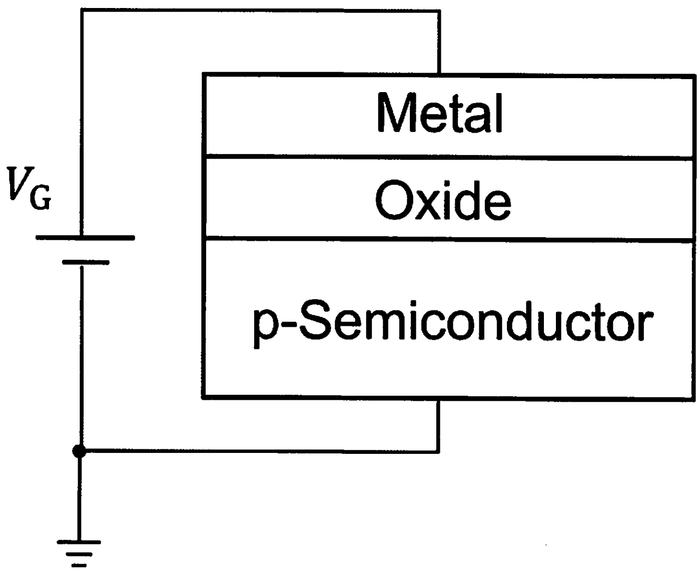

Figure 6-1 Metal/oxide/p-type semiconductor (MOS) structure.

令和８年度８月実施（令和７年度実施）

出題意図

問題１(1) 変数変換、連立方程式、複素積分等の基本事項が理解できるかを問う。

問題 1(2) 2 次正方行列の対角化と固有値問題に関する基本的な理解を問う。

問題２ 慣性モーメントについての基本事項が理解できるかを問う。

問題３ 静電場についての基本事項が理解できるかを問う。

問題４シュレディンガー方程式の変数変換が理解でき、第一励起状態への応用ができるかを問う。

問題５ 光学にかかわる基本事項が理解できるかを問う。

問題６ MOS ダイオードにかかわる基本事項が理解できるかを問う。

令和8年度入学

筑波大学大学院入学試験

１月実施

理工情報生命学術院　数理物質科学研究群

応用理工学学位プログラム

電子・物理工学サブプログラム　試験問題

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Applied Physics

Problem 1 (140 points) Mathematics Part (1)

Note: Problem 1 has two parts, (1) and (2). Answer all of them.

(1) Answer the following questions (a) and (b). Let x, y, a be real numbers [実数], n be a natural number [自然数], π be the circular constant [円周率], i be the imaginary unit, and e be the base of the natural logarithm.

(a) Answer the following questions.

(i) Solve the following equation.

$$\sin^{-1} x = \tan^{-1} \sqrt{8}$$

(ii) Find the value of A in the following equation, where $\bar{A}$ is the complex conjugate of A.

$$\sin\left(\frac{\pi}{3} - i\right) = Ae + \bar{A}\, e^{-1}$$

(iii) Find the area of following region of D.

$$D = \{(x, y) | x^2 - 2xy + 5y^2 \leq a^2\} \quad (a > 0)$$

(b) Evaluate the following integrals.

(i) $$\int_0^\infty x^n e^{-x} dx$$

(ii) $$\iint_D \frac{dxdy}{\sqrt{1 - x^2 - y^2}}, \ D = \{(x, y) | x^2 + y^2 < 1\}$$

(continued to part (2) on the next page)

Applied Physics

Problem 1 (140 points) Mathematics Part (2)

Note: Problem 1 has two parts, (1) and (2). Answer all of them.

(2) Consider the differential equation

$$\frac{d^2}{dt^2}\boldsymbol{r}(t) = A\boldsymbol{r}(t),$$

where $A = \begin{pmatrix} a & c & 0 \\ c & b & c \\ 0 & c & a \end{pmatrix}, \boldsymbol{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$. Here $a, b,$ and c are real number constants, t is a real number variable, and $x(t), y(t),$ and $z(t)$ are real functions. Answer the following questions.

Given that $a = -1, b = -2,$ and $c = 1$, answer the following questions (a) - (d).

(a) Find eigenvalues, λ_1, λ_2, and λ_3 of A, where $\lambda_1 \le \lambda_2 \le \lambda_3$.

(b) Find the normalized eigenvectors, $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, and $\boldsymbol{u}_3$ of A, where $A\boldsymbol{u}_j = \lambda_j \boldsymbol{u}_j$ $(j = 1, 2, 3)$.

(c) Find a diagonal matrix D, an orthogonal matrix P, and P^{-1}, where A is diagonalized as $P^{-1}AP = D$.

(d) Given that $\boldsymbol{R}(t) = P^{-1}\boldsymbol{r}(t)$. Prove $\frac{d^2}{dt^2}\boldsymbol{R}(t) = D\boldsymbol{R}(t)$.

Applied Physics

Problem 2 (120 points) Mechanics

Consider the moment of inertia (慣性モーメント) around the central axis of solid cylinder A and hollow (中空の) cylinder B shown in Fig. 2-1. The radius of solid cylinder A is R and its length is L. The density of solid cylinder A is uniform, ρ_A. The outer and inner radii of hollow cylinder B are R and r, respectively, and its length is L. The masses of solid cylinder A and hollow cylinder B are both M. Answer the following questions, assuming that air resistance can be neglected.

(1) As preparation for calculating the moment of inertia of solid cylinder A, consider a cylindrical shell (殻) of density ρ_A, radius x, length L and thickness δx as shown in Fig. 2-2, where δx is infinitely small (無限小). Express the mass of this cylindrical shell δm using L, ρ_A, x, and δx.

(2) Express the moment of inertia around the central axis of solid cylinder A, I_A, using M and R. Show the process of calculation.

(3) Express the moment of inertia of hollow cylinder B around the central axis, I_B, using M, R and r. The density of hollow cylinder B is uniform.

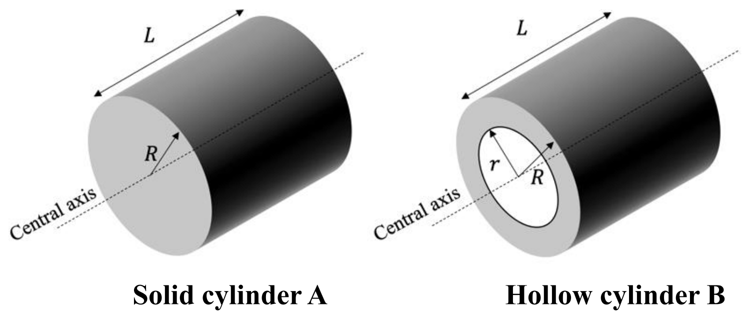

Fig. 2-1

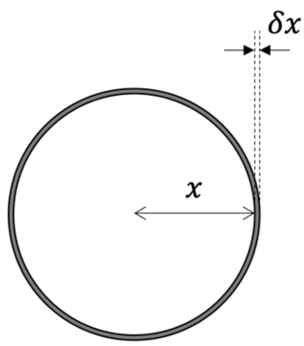

Fig. 2-2

As shown in Fig. 2-3, solid cylinder A and hollow cylinder B are at the same height from the floor, and are stationary on a slope with an angle θ from the horizontal floor. The central axes of solid cylinder A and hollow cylinder B are set to be perpendicular to the direction parallel to the slope. Suppose that at time $t = 0$, solid cylinder A and hollow cylinder B start to move in the direction parallel to the slope with an initial velocity of zero. Answer the following questions.

(*continued to the next page*)

(4) Consider the case that there is no friction on the slope, and solid cylinder A and hollow cylinder B slide without rolling. The time when they reach the floor is t_{A} and t_{B}, respectively. Show the relation between t_{A} and t_{B}, by using the equal sign (=) or inequality sign (<, >).

(5) Consider the case that there is friction on the slope, and solid cylinder A and hollow cylinder B roll down the slope without sliding. Show the equation of motion for solid cylinder A, and express the magnitude of the acceleration a (加速度) of the center of gravity (重心) of cylinder A, using g, θ, I_{A}, M, and R. g is the acceleration of gravity.

Next, as shown in Fig. 2-4, consider solid sphere C (with radias R and mass M), composed of uniform density, in addition to solid cylinder A and hollow cylinder B on the slope with friction. All other conditions are the same as in (5).

(6) The time when solid cylinder A, hollow cylinder B, and solid sphere C reach the floor is t'_{A}, t'_{B}, and t'_{C}, respectively. Show the relationship between t'_{A}, t'_{B}, and t'_{C} by using the equal sign (=) or inequality sign (<, >). Here the the moment of inertia of solid sphere C is give by $I_{\mathrm{C}} = \frac{2}{5}MR^2$.

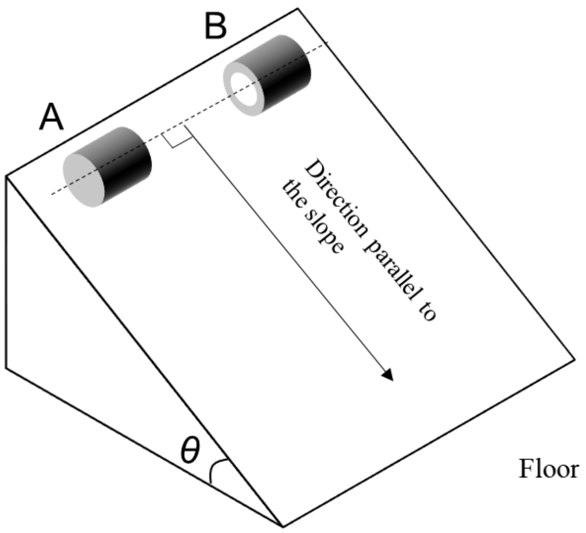

Fig. 2-3

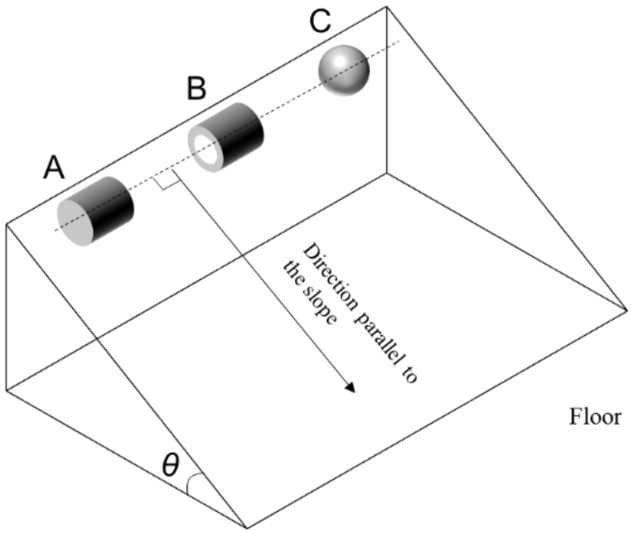

Fig. 2-4

Applied Physics

Problem 3 (120 points) Electromagnetism

Answer the following questions.

(1) Consider a capacitor made of two coaxial cylindrical electrodes of radii a and b $(a < b)$, as shown in Fig. 3-1. Both cylindrical electrodes have the same length L, and L is assumed to be sufficiently larger than a and b. Therefore, edge effects can be neglected. In addition, each electrode is assumed to be sufficiently thin. Assume that the region between the two cylindrical electrodes is filled with a dielectric material with permittivity ε (誘電率). Then the inner electrode is given a charge of λ (> 0) per unit length, and the outer electrode is given a charge of $-\lambda$ per unit length. Let the magnitude of the electric flux density, that of the electric field, and the electrostatic potential at a point located at a distance r $(a < r < b)$ from the central axis be denoted by $D(r)$, $E(r)$, and $V(r)$, respectively. In the dielectric material, the relation $D(r) = \varepsilon E(r)$ holds.

(a) Consider a coaxial cylindrical surface that passes through a point at a distance r from the central axis of the capacitor. Gauss's law for the cylindrical surface is given by $D(r)S = L\lambda$. Express S in terms of L and r.

(b) Express $D(r)$ and $E(r)$ in terms of necessary ones among λ, r, and ε.

(c) Express $V(r)$ in terms of λ, b, r, and ε. Assume that $V(b) = 0$.

(d) Express the capacitance of this capacitor in terms of L, a, b, and ε.

(2) Suppose that a steady current of magnitude I is flowing in a circular loop of radius a with its center at the origin O in vacuum, as shown in Fig. 3-2. Consider the magnetic flux density at position $\boldsymbol{r}$ on the line l passing through the center O and perpendicular to the plane of the circular loop. Denote the distance between the position $\boldsymbol{r}$ and O by $r = |\boldsymbol{r}|$. Let μ_0 be the magnetic permeability in vacuum.

(a) According to the Biot-Savart law, the magnetic flux density at position $\boldsymbol{r}$ produced by a current element $I\boldsymbol{t}(\boldsymbol{r}')\Delta s$ flowing on the small segment of length Δs at position $\boldsymbol{r}'$, shown in Fig. 3-2, is given by $\Delta\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{I\boldsymbol{t}(\boldsymbol{r}')\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\Delta s$. Here $\boldsymbol{t}(\boldsymbol{r}')$ is the unit vector pointing in the direction of the current in the current element. Express the magnitude $\Delta B(\boldsymbol{r}) = |\Delta\boldsymbol{B}(\boldsymbol{r})|$ in terms of $\mu_0, I, a, r,$ and Δs.

(*continued to the next page*)

(b) Express the component of $\Delta\boldsymbol{B}(\boldsymbol{r})$ in (a) that is parallel to the line l in terms of $\mu_0, I, a, r,$ and Δs.

(c) Express the magnitude of the magnetic flux density at position $\boldsymbol{r}$ produced by the entire current flowing in the circular loop in terms of μ_0, I, a, and r.

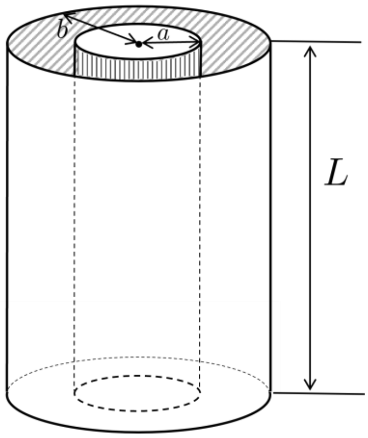

Fig. 3-1

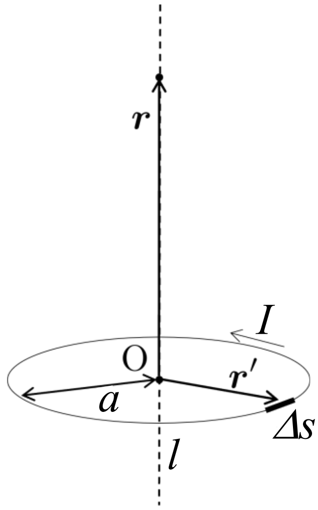

Fig. 3-2

Applied Physics

Problem 4 (120 points) Quantum Mechanics

Consider a particle of mass m confined in a one-dimensional quantum well with infinite barrier height. The potential $V(x)$ is given as follows.

$$V(x) = \begin{cases} 0 & (0 < x < a) \\ \infty & (x \leq 0, a \leq x) \end{cases}$$

When the time-independent Schrödinger equation is solved under this condition, the normalized eigenfunctions [正規化された固有関数] $\varphi_n(x)$ can be expressed as follows with the quantum number n.

$$\varphi_n(x) = \begin{cases} \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi}{a}x\right) & (0 < x < a) \\ 0 & (x \leq 0, a \leq x) \end{cases}$$

Let $\hbar$ denote the reduced Planck constant (Dirac constant). Answer the following questions.

(1) Find the energy eigenvalue ε_n of $\varphi_n(x)$.

(2) The momentum operator [運動量演算子] is given by $\hat{p} = -i\hbar\frac{\partial}{\partial x}$. For $\varphi_n(x)$, calculate the expectation values [期待値] $\bar{p}$ and $\overline{p^2}$.

Next, consider a time-dependent wave function $\psi(x,t)$. Here, the initial state ($t = 0$) of $\psi(x,t)$ is defined by a linear combination of $\varphi_1(x)$ and $\varphi_2(x)$ as shown below, where A is a constant.

$$\psi(x,0) = A\{\varphi_1(x) + \varphi_2(x)\}$$

(3) Determine the constant A by normalizing the wave function $\psi(x,0)$, where $A > 0$.

(4) Express the wave function $\psi(x,t)$ in terms of a, ε_1, and ε_2.

(5) Let $S(x,t)$ denote the probability current [確率の流れ]. Calculate $S\left(\frac{a}{2},t\right)$, using the fact that $S(x,t) = \frac{1}{m}\mathrm{Re}\left[\psi^*(x,t)\left(-i\hbar\frac{\partial\psi(x,t)}{\partial x}\right)\right]$.

(Continued to the next page)

(6) Find the probability that the particle with the wave function $\psi(x,t)$ exists in the range of $-\infty \leq x \leq \frac{a}{2}$. Then find its time derivative [時間微分]. Compare the calculated time derivative with the $S\left(\frac{a}{2},t\right)$ obtained in (5), and explain their relationship physically.

Applied Physics

Problem 5 (120 points) Optics

Answer the question about geometrical optics [幾何光学] and wave optics [波動光学].

(1) Answer whether each of the following terms belongs to geometrical or wave optics. List all terms on the answer sheet, and select (write) “geometrical optics (or GO)” or “wave optics (or WO)” for each term. For terms relates to both geometrical and wave optics, answer “BOTH.”
Terms: “ray [光線],” “diffraction [回折],” “interference [干渉],” “image formation [結像]”, “polarization [偏光],” “wavefront [波面],” “equi-phase plane [等位相面],” “Seidel’s five types of aberration [ザイデルの5収差],” “aberration represented by Zernike polynomial [Zernike 多項式による収差の表現],” “Fermat’s principle [フェルマーの原理].”

(2) Explain what refractive index is using some or all of the following terms; wavelength, speed of light, and vacuum. You may use equation(s) if necessary.

(3) We consider light incident from a material with refractive index n_1 to a material with refractive index of n_2 as shown in Figs. 5-1 and 5-2. Here the interface between the two materials is flat. Both Figs. 5-1 and 5-2 illustrate the concept of “refraction,” one based on geometrical optics and the other based on wave optics. (i) Answer whether each illustration is based on geometrical or wave optics. (ii) State what A in Fig. 5-1 and B in Fig. 5-2 represent.

(4) Using the term “refractive index,” explain what “optical path length” is.

(5) Refer to Fig. 5-2 and express the relationship between the angle of incidence θ_1 and the angle of refraction θ_2 using the refractive indices n_1 and n_2.

(6) To verify the relationship in (5), select one of the following methods: “Fermat's principle,” “wavefront continuity,” or “Huygens' secondary waves [ホイヘンスの2次波].” Outline how you would prove it. You may use figures or illustrations if necessary.

(7) Prove Snell’s law using the method selected in (6).

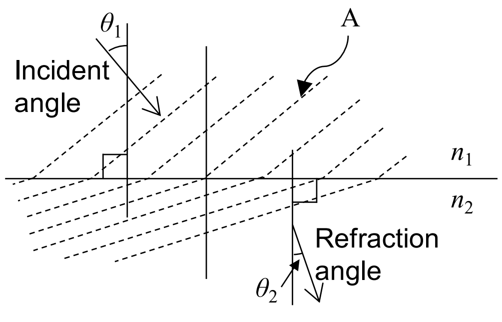

Fig. 5-1

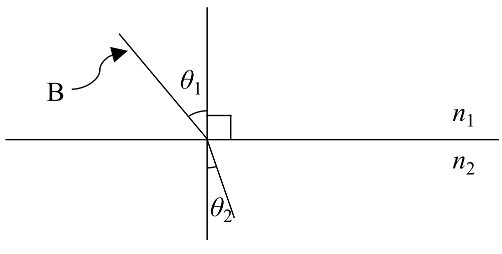

Fig. 5-2

Applied Physics

Problem 6 (120 points) Semiconductor Physics

Consider a pn homojunction between an n-type semiconductor with impurity concentration N_D and a p-type semiconductor with impurity concentration N_A, as shown in Fig. 6-1. Assume that the impurities are uniformly doped in both the p-type and n-type semiconductors and are fully ionized. The depletion layer widths in the n-type region and p-type region are denoted by W_n and W_p, respectively. Let q be the elementary charge, V_d the diffusion potential (拡散電位), ε the permittivity (誘電率), k_B the Boltzmann constant, and T the absolute temperature. Answer the following questions.

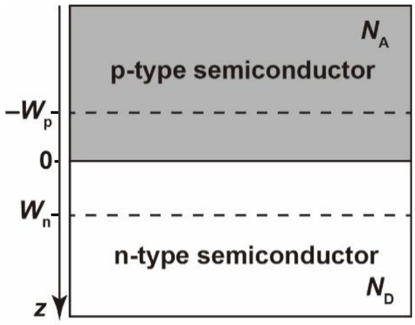

Fig. 6-1 pn junction

1. What elements (元素) should be added as dopants to crystalline silicon (c-Si) ? List two elements necessary to form p-type and n-type c-Si, respectively, and explain the reasons for their selection.
2. Draw the energy band diagram of the pn junction in thermal equilibrium. Clearly indicate the Fermi level E_F, conduction band minimum E_C, valence band maximum E_V, W_n, W_p, and V_d.
3. Describe the Poisson's equation for the depletion region within the p-type semiconductor.
4. Describe the potential $\phi(z)$ in the depletion regions within both p-type and n-type semiconductors and draw the potential $\phi(z)$ as a function of depth z throughout the whole pn junction region under the assumption of $\phi(-W_p) = 0$.
5. Express V_d using W_n, W_p, N_D, and N_A.
6. Express W_n and W_p using V_d, N_D, and N_A.
7. The depletion region can be regarded as a dielectric layer with dielectric constant ε and total thickness $W\,(=W_n + W_p)$, which acts like a parallel-plate capacitor. Express the capacitance per unit area C of this depletion region using V_d, N_D, and N_A.
8. When an external voltage V is applied across the pn junction, draw the graph on the relationship between $1/C^2$ and V. Note that a positive sign of voltage indicates that a forward bias is applied across the pn junction. Also, explain the physical meaning of the intercept of this graph when extrapolated to $1/C^2 = 0$.

令和 8 年度 1 - 2 月実施（令和 7 年度実施）

出題意図

問題 1 (1) 複素数、部分積分、座標変換等の基本事項が理解できるかを問う。

問題 1(2) 3 次正方行列の対角化と固有値問題に関する基本的な理解を確認し、線形微分方程式への応用ができるかを問う。

問題 2 慣性モーメントについての基本事項が理解できるかを問う。

問題 3 静電場、静磁場についての基本事項が理解できるかを問う。

問題 4 シュレディンガー方程式の基本が理解でき、期待値や確率の流れなどへの応用ができるかを問う。

問題 5 光学にかかわる基本事項が理解できるかを問う。

問題 6 半導体 pn 接合にかかわる基本事項が理解できるかを問う。